Broadband Complex Permittivity Measurements of Dielectric Substrates using a Split-Cylinder Resonator

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***Abstract*—We discuss a theoretical model describing the split-cylinder resonator for nondestructive measurement of a dielectric substrate's relative permittivity and loss tangent. This improved model properly accounts for the fringing electric and magnetic fields in the dielectric substrate. Previously, the split-cylinder resonator has been used for single-frequency permittivity and loss tangent measurements using only the fundamental TE_{011} resonant mode. By including higher-order TE_{0np} modes, we demonstrate how to measure the relative permittivity and loss tangent of dielectric substrates over an extended frequency range. We validated the new model by measuring the permittivity and loss tangent of fused-silica substrates from 10 to 50 GHz and comparing with results obtained with a circular-cylindrical cavity, a dielectric-post resonator, and several split-post resonators.**[1]

***Index Terms*—cavity resonators, dielectric measurements, loss tangent, permittivity.**

I. Introduction

The split-cylinder resonator technique is a nondestructive method for measuring the permittivity and loss tangent of low-loss dielectric substrates. Originally proposed by Kent [1][2], this method employs a circular-cylindrical cavity that is separated into two halves, as shown in Fig. 1. A sample is placed in the gap between the two halves of the cavity sections. A coupling loop in each waveguide section excites a TE_{0np} resonance, and from measurements of the resonant frequency and quality factor, the permittivity and loss tangent of the sample can be determined.

The advantage of the split-cylinder method is that the sample needs only to be planar and to extend sufficiently far beyond the walls of the two cylindrical waveguide sections. No other sample machining is necessary, making this method attractive for accurate, nondestructive measurements of low-loss substrates. Unfortunately, reduced sample preparation comes at the cost of a complicated theoretical model. In order to produce accurate relative permittivity and loss tangent results, the model must accurately account for the fringing electric and magnetic fields that extend into the sample region beyond the cylindrical waveguide regions.

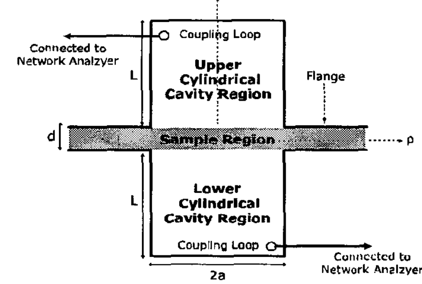

Fig. 1. Cross-sectional diagram of a split-cylinder resonator.

Previously [3], we developed a theoretical model employing Hankel transforms that rigorously accounted for these fringing fields. We derived a resonance condition for the split-cylinder resonator that allowed us to calculate the sample permittivity. Although accurate, this model is computationally intensive, so we have developed a new theoretical model, based on the mode-matching method. Using this model, we derive equations for calculating the relative permittivity and loss tangent of a sample [4]. We summarize this theoretical model and demonstrate how to use the resulting equations to calculate the sample's relative permittivity ϵ_s' and loss tangent $\tan\delta$.

In the past, only the fundamental TE_{011} resonant mode of the split-cylinder resonator has been used to measure the permittivity and loss tangent. However, the mode-matching model we present is also valid for higher-order TE_{0np} resonant modes. From measurements of the TE_{0np} resonant frequency and quality factor, we show how to calculate the relative permittivity and loss tangent of the substrate under test over a frequency range of 10 to 50 GHz. In order to verify the accuracy of the results, we compare measurements on fused-silica substrates using a split-cylinder resonator to measurement data obtained with

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a circular-cylindrical cavity, dielectric-post resonator, and several split-post resonators [5].

II. Relative Permittivity

In our initial model for calculating the sample permittivity, we assumed that the sample extended to infinity in the radial direction [3]. However, if the fringing electric and magnetic fields decrease as a function of ρ in the sample region, we may assume that a perfect conductor exists at some sufficiently large $\rho = b$. By introducing this boundary, we enclose the entire split-cylinder resonator fixture, thereby enabling us to derive a valid resonance condition using the mode-matching method without introducing a systematic error in the calculation of the sample's relative permittivity and loss tangent.

For a TE_{0np} resonant mode, we determined the transverse electric and magnetic fields in the upper cylindrical-cavity region from Maxwell's equations with enforcement of the boundary conditions on the upper waveguide end-plate and walls:

$$E_{\phi_u} = \sum_{n=1}^{N_u} A_n U_n J_1(h_{n_u}\rho) \sin\left[p_{n_u}\left(L + \frac{d}{2} - z\right)\right], \quad (1)$$

$$H_{\rho_u} = -\frac{1}{j\omega\mu_0} \sum_{n=1}^{N_u} A_n U_n p_{n_u} J_1(h_{n_u}\rho) \cos\left[p_{n_u}\left(L + \frac{d}{2} - z\right)\right], \quad (2)$$

where $p_{n_u}^2 = k_u^2 - h_{n_u}^2$, $k_u^2 = \omega^2\mu_0\epsilon_0\epsilon_a'$, ϵ_a' is the relative permittivity of air, $h_{n_u} = \frac{j_{1,n}}{a}$, J_1 is the Bessel function of the first kind of order one, $j_{1,n}$ is the nth zero of J_1, A_n are constants to be determined, and N_u is the total number of modes included in the upper cylindrical-cavity region. Additional geometrical parameters are given in Fig. 1. To improve the conditioning of the matrix $\mathbf{Z}$ in (16), we have included the factors $U_n = p_{N_u}/\cosh\left[\mathrm{Im}(p_{n_u})L\right]$ in (1) and (2).

Since the sample region is also enclosed by a perfect conductor at $\rho = b$, we similarly express the transverse electric and magnetic fields in the sample region:

$$E_{\phi_s} = \sum_{n=1}^{N_s} B_n V_n J_1(h_{n_s}\rho) \cos(p_{n_s} z), \quad (3)$$

$$H_{\rho_s} = -\frac{1}{j\omega\mu_0} \sum_{n=1}^{N_s} B_n V_n p_{n_s} J_1(h_{n_s}\rho) \sin(p_{n_s} z), \quad (4)$$

where $p_{n_s}^2 = k_s^2 - h_{n_s}^2$, $k_s^2 = \omega^2\mu_0\epsilon_0\epsilon_s'$, ϵ_s' is the relative permittivity of the sample, $h_{n_s} = \frac{j_{1,n}}{b}$, B_n are constants to be determined, and N_s is the total number of modes included in the sample region. In this case, we similarly included the terms $V_n = p_{N_s}/\cosh\left[\mathrm{Im}(p_{n_s})\frac{d}{2}\right]$ in (3) and (4) to improve the conditioning of the matrix $\mathbf{Z}$ in (16).

We derive a resonance condition by enforcing the boundary conditions for the transverse electric and magnetic field. The tangential electric and magnetic fields are continuous at $z = d/2$:

$$E_{\phi_u}\left(z = \frac{d}{2}\right) = E_{\phi_s}\left(z = \frac{d}{2}\right) \quad 0 \le \rho \le b, \quad (5)$$

$$H_{\rho_u}\left(z = \frac{d}{2}\right) = H_{\rho_s}\left(z = \frac{d}{2}\right) \quad 0 \le \rho \le a. \quad (6)$$

Substituting the series electric and magnetic field expressions (1-4) into (5) and (6), multiplying each side by the electric or magnetic field [H_{ρ_s} in the case of (5), E_{ϕ_u} in the case of (6)], integrating over the appropriate cross-section, and employing orthogonality of the modes [6], we obtain two systems of equations:

$$\mathbf{QA} = \mathbf{RB} \quad (7)$$

and

$$\mathbf{SA} = \mathbf{PB}, \quad (8)$$

where

$$Q_{mn} = A_n U_n \frac{a h_{nu}}{h_{ms}^2 - h_{nu}^2} J_1(h_{ms}a) J_0(h_{nu}a) \sin(p_{nu}L), \quad (9)$$

$$R_{mm} = B_m V_m \frac{b^2}{2} J_0^2(h_{ms}b) \cos\left(p_{ms}\frac{d}{2}\right), \quad (10)$$

$$S_{nn} = A_n U_n p_{nu} \frac{a^2}{2} J_0^2(h_{nu}a) \cos(p_{nu}L), \quad (11)$$

and

$$P_{nm} = B_n V_m p_{Ns} \frac{a p_{ms} h_{nu}}{h_{ms}^2 - h_{nu}^2} J_1(h_{ms}a) J_0(h_{nu}a) \sin\left(p_{ms}\frac{d}{2}\right). \quad (12)$$

Both $\mathbf{R}$ and $\mathbf{S}$ are diagonal matrices.

The system of equations represented by (7) and (8) can be rewritten as

$$[\mathbf{Z}][\mathbf{X}] = \mathbf{0}, \quad (13)$$

where

$$[\mathbf{Z}] = \begin{bmatrix} \mathbf{Q} & -\mathbf{R} \\ \mathbf{S} & -\mathbf{P} \end{bmatrix} \quad (14)$$

and

$$[\mathbf{X}] = \begin{bmatrix} \mathbf{A} \\ \mathbf{B} \end{bmatrix}. \quad (15)$$

The resonance condition follows from the fact that this linear system of equations has a nontrivial solution only if

$$\det[\mathbf{Z}] = 0. \quad (16)$$

We can use (16) to iteratively calculate either the resonant frequency of the split-cylinder cavity given a known sample permittivity ϵ_s', or the sample permittivity given a measured resonant frequency f.

III. Loss Tangent

In order to discuss the measurement of the sample loss tangent $\tan\delta$, we must first examine the definition of the quality factor Q of the split-cylinder resonator when a sample is present:

$$Q = \frac{\omega(W_a + W_s)}{P_w + P_e + P_f + P_s}, \quad (17)$$

where W_a and W_s are the time-average energies stored in the cylindrical cavity and sample regions, and P_w, P_e, P_f, and P_s are the powers dissipated in the cylindrical cavity walls, endplates, flange, and sample respectively. We have ignored the power dissipated in the coupling loops because we ensured that the resonance was very weakly coupled. At the peak of the resonance, the magnitude of the scattering parameter S_{21} was kept below -50 dB by adjustment of the coupling loops. When we calculate the sample permittivity using (16), we also determine the coefficients A_n and B_n, thereby allowing us to write the energy-stored terms as

$$W_s = \epsilon_0\epsilon_s' \int_{z=0}^{\frac{d}{2}} \int_{\rho=0}^{b} \int_{\phi=0}^{2\pi} |E_{\phi_s}|^2 \rho \, d\phi \, d\rho \, dz, \quad (18)$$

$$W_a = \epsilon_0\epsilon_a' \int_{z=\frac{d}{2}}^{L} \int_{\rho=0}^{a} \int_{\phi=0}^{2\pi} |E_{\phi_u}|^2 \rho \, d\phi \, d\rho \, dz, \quad (19)$$

and the power-dissipation terms as

$$P_e = R_s \int_{\rho=0}^{a} \int_{\phi=0}^{2\pi} |H_{\rho_u}|^2 \rho \, d\phi \, d\rho \Big|_{z=L}, \quad (20)$$

$$P_w = R_s \int_{z=\frac{d}{2}}^{L} \int_{\phi=0}^{2\pi} |H_{z_u}|^2 \rho \, d\phi \, dz \Big|_{\rho=a}, \quad (21)$$

$$P_f = R_s \int_{\rho=a}^{b} \int_{\phi=0}^{2\pi} |H_{\rho_s}|^2 \rho \, d\phi \, d\rho \Big|_{z=\frac{d}{2}}, \quad (22)$$

and

$$P_s = \tan\delta \, \omega\epsilon_0\epsilon_s' \int_{\rho=0}^{b} \int_{z=0}^{\frac{d}{2}} \int_{\phi=0}^{2\pi} |E_{\phi_s}|^2 \rho \, d\phi \, d\rho \, dz. \quad (23)$$

Given that ϵ_s' is calculated from the measured resonant frequency, the two remaining unknown variables used to determine the energy stored and power dissipated are the surface resistivity R_s of the cylindrical waveguide sections and the loss tangent $\tan\delta$ of the sample. We obtain R_s from a measurement of the quality factor of the empty split-cylinder resonator, where the gap between the cylindrical waveguide sections is closed. Then, after measuring the quality factor Q of the split-cylinder when the sample is present, we solve (17) for the sample loss tangent $\tan\delta$.

IV. Measurement Results

We used two split-cylinder resonators to measure the complex permittivity of two fused-silica substrates over a frequency range 10 - 50 GHz. The first split-cylinder resonator, whose TE_{011} resonant frequency is 10 GHz with no sample, had dimensions $2a$ = 38.1 mm and L = 25.3 mm. The resonator was constructed from oxygen-free copper. A small hole in the waveguide wall allows the passage of the coupling loop. The second split-cylinder resonator, whose TE_{011} resonant frequency is 35 GHz with no sample, had dimensions $2a$ = 13.18 mm and L = 3.51 mm. The resonator was constructed from silver-plated brass and also contained small holes in each waveguide section to accommodate the coupling loops.

Two fused-silica samples machined from the same lot were measured with the two split-cylinder resonators. The sample for the larger split-cylinder resonator was 55 mm square and 0.81 mm thick. The sample for the smaller resonator was 25 mm square and 0.28 mm thick. Each sample was placed between the two waveguide sections of the split-cylinder resonator, and the resonance curve for the TE_{011} mode was examined on an automatic network analyzer. From the resonance curve, we obtained the resonance frequency f and the quality factor Q [7]. From these two measured quantities and the geometrical dimensions of the split-cylinder resonator and sample, we calculated the sample relative permittivity using (16) and the sample loss tangent using (17).

We noted earlier that equation (16) could be used to calculate the resonant frequency of the split-cylinder resonator, given the substrate's relative permittivity and thickness and the dimensions of the split-cylinder resonator. Using the value of the relative permittivity calculated from the TE_{011} resonance, we determined the frequencies of the higher-order TE_{011} resonances by calculating $\det[\mathbf{Z}]$ as a function of frequency. The first zero of this function corresponds to the TE_{011} mode while the other zeroes correspond to the higher-order TE_{0np} modes. Observing where these frequencies occur before attempting to measure them with a network analyzer is important because as the frequency increases, other TM and TE resonant modes, besides those in the TE_{0np} family, are excited.

After correctly identifying the higher-order modes, we measured the resonant frequency and quality factor of each using a network analyzer. In some cases, the TE_{0np} mode may be close in frequency to other resonant modes, distorting the resonance curve. We need to use these modes with caution since this distortion may cause errors in the calculation of the resonant frequency and quality factor. Once the resonant frequency and quality factor were

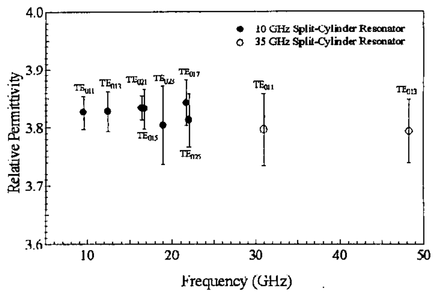

Fig. 2. Relative permittivity measurements of fused silica using several split-cylinder resonator modes.

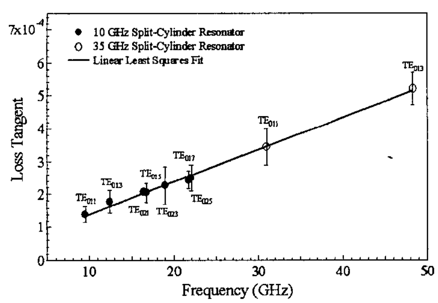

Fig. 3. Loss tangent measurements of fused silica using several split-cylinder resonator modes.

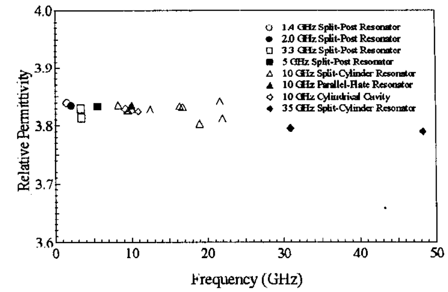

Fig. 4. Broadband relative permittivity measurements of fused silica using various measurement methods.

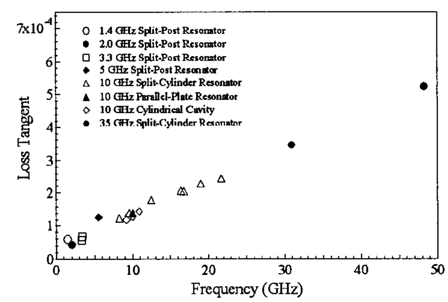

Fig. 5. Broadband loss tangent measurements of fused silica using various measurement methods.

accurately measured, we calculated the relative permittivity and loss tangent of the substrate using (16) and (17). Figure 2 shows the measured relative permittivity of fused silica using our two split-cylinder resonators. As expected, the permittivity is relatively flat over the frequency range. Fig. 3 shows the measured loss tangent of fused silica measured with the two split-cylinder resonators. We noted an increase in the loss tangent as a function of frequency, and when we performed a least squares fit to the data we found the increase in the loss tangent to be linear over the frequency range 10 - 50 GHz.

Finally, to verify the accuracy of the measurements on fused silica, we machined additional samples for testing in a circular-cylindrical cavity, a dielectric-post resonator, and several split-post resonators. In Figures 4 and 5 we compare our split-cylinder resonator measurements of relative permittivity and loss tangent to measurements made using these other methods. We found good agreement between all the methods for both relative permittivity and loss tangent over the frequency range from 1 - 50 GHz.

V. Conclusions

Using the mode-matching method, we developed a theoretical model for the split-cylinder resonator method. The model includes not only the TE_{011} resonant mode, but also the higher-order TE_{0np} modes that occur at higher frequencies. Using measurements of the resonant frequencies and quality factors of the higher-order TE_{0np} modes, we measured the relative permittivity and loss tangent of fused silica over a frequency range of 10 to 50 GHz and found them to agree with measurements made in a circular-cylindrical cavity, dielectric-post resonator, and several split-post resonators.

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